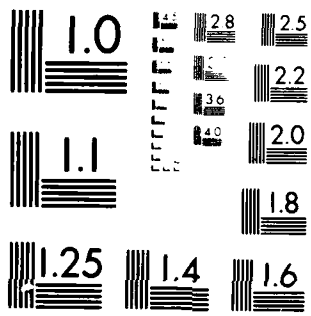
AD-A122 230 A GENERAL PROGRAM FOR PREDICTING RIGID-AIRCRAFT GUST RESPONSE(U) AERONAUTICAL RESEARCH LABS MELBOURNE (AUSTRALIA) R A FEIK JAN 82 ARL-AERO-NOTE-407 1/1

UNCLASSIFIED F/G 1/3 NL

END
DATE
FILMED
1 83
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

(12)

ARL-AERO-NOTE-407

AR-002-331

DEPARTMENT OF DEFENCE

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

AERONAUTICAL RESEARCH LABORATORIES

MELBOURNE, VICTORIA

AD A122230

AERODYNAMICS NOTE 407

A GENERAL PROGRAM FOR PREDICTING RIGID-AIRCRAFT GUST RESPONSE

by

R. A. FEIK

Approved for Public Release

COPY No

JANUARY, 1982

AR-002-331

DEPARTMENT OF DEFENCE
DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION
AERONAUTICAL RESEARCH LABORATORIES

AERODYNAMICS NOTE 407

A GENERAL PROGRAM FOR PREDICTING RIGID-AIRCRAFT GUST RESPONSE

by

R. A. FEIK

SUMMARY

A Fortran program has been developed for rigid-aircraft gust response calculations. Equations of motion for the aircraft dynamics are defined in the program and may be augmented to include flight control systems. Input data are used to described a particular aircraft configuration, aerodynamic data and flight conditions. The program calculates the gust response transfer functions and combines them with gust spectra to obtain the output response spectra. The relatively simple gust model included may be modified or extended readily. The transfer functions may also be used to obtain time-domain responses to discrete gusts. Examples are given showing the effects on gust response of variations in configuration, aerodynamics and control systems.

POSTAL ADDRESS: Chief Superintendent, Aeronautical Research Laboratories, Box 4331, P.O., Melbourne, Victoria, 3001, Australia.

DOCUMENT CONTROL DATA

1. a. AR No. AR-002-331	1. b. Establishment No. ARL-Aero-Note-407	2. Document Date January, 1982	3. Task No. DST 79/105

Title
A GENERAL PROGRAM FOR PREDICTING RIGID-AIRCRAFT GUST RESPONSE

5. Security
a. document
Unclassified
b. title c. abstract
Unclass. Unclass.

6. No. pages
13

7. No. Refs
6

8. Author(s)
R. A. Feik

9. Downgrading Instructions
—

10. Corporate Author and Address
Aeronautical Research Laboratories

11. Authority (*as appropriate*)
a. Sponsor c. Downgrading
b. Security d. Approval

12. Secondary Distribution (*of this document*)
Approved for public release

Overseas enquirers outside stated limitations should be referred through ASDIS, Defence Information Services Branch, Department of Defence, Campbell Park, CANBERRA, ACT 2601

13. a. This document may be ANNOUNCED in catalogues and awareness services available to . . .
No limitations

13. b. Citation for other purposes (*i.e. casual announcement*) may be unrestricted

14. Descriptors

Gust response	Dynamic response
Aircraft	Aerodynamic loads
Gusts	Computer programs
Power spectra	Aerodynamic characteristics
Transfer functions	GUSTR (Computer program)

15. COSATI Group
0103
0902

16. Abstract

A Fortran program has been developed for rigid-aircraft gust response calculations. Equations of motion for the aircraft dynamics are defined in the program and may be augmented to include flight control systems. Input data are used to describe a particular aircraft configuration, aerodynamic data and flight conditions. The program calculates the gust response transfer functions and combines them with gust spectra to obtain the output response spectra. The relatively simple gust model included may be modified or extended readily. The transfer functions may also be used to obtain time-domain responses to discrete gusts. Examples are given showing the effects on gust response of variations in configuration, aerodynamics and control systems.

17. Imprint
Aeronautical Research Laboratories, Melbourne

18. Document Series and Number	19. Cost Code	20. Type of Report and Period Covered
Aerodynamics Note 407	52 7730	

21. Computer Programs Used
GUSTR; DETPOL. FOR; POLSUB.FOR—all Fortran

22. Establishment File Ref(s)

CONTENTS

NOTATION

a_0, a_1, a_2	Coefficients defining element of $\mathbf{A}(s)$, Equation (25)
$\mathbf{A}(s)$	Matrix defining system model, Equation (1)
$\mathbf{A}_1(s), \mathbf{A}_2(s)$	Longitudinal and lateral aircraft models
$A0, A1, A2[I, J]$	Two-dimensional arrays specifying elements of $\mathbf{A}$
$\mathcal{R}$	Ratio of span/mean aerodynamic chord, $b/\bar{c}$
b	Span
b_0, b_1, b_2	Coefficients defining elements of $\mathbf{B}(s)$, Equation (25)
$\mathbf{B}(s)$	Matrix of gust input transfer functions, Equation (1)
$\mathbf{B}_1(s), \mathbf{B}_2(s)$	Longitudinal and lateral versions of $\mathbf{B}$
$B0, B1, B2[I]$	One-dimensional arrays specifying columns of $\mathbf{B}$
$\bar{c}$	Mean aerodynamic chord
C_D, C_L, C_Y, C_T	Drag, lift, sideforce and thrust coefficients, Force/$\frac{1}{2}\rho V^2 S$
C_l, C_n	Rolling and yawing moment coefficients, Moment/$\frac{1}{2}\rho V^2 Sb$
C_m	Pitching moment coefficient, Moment/$\frac{1}{2}\rho V^2 S\bar{c}$
C_W	Aircraft weight coefficient, mg/$\frac{1}{2}\rho V^2 S$
$C_{D_V}, C_{L_V}, C_{m_V}, C_{T_V}$	Non-dimensional derivatives w.r.t. V, $\partial C_D/\partial(V/V_e)$, etc.
$C_{D_\alpha}, C_{L_\alpha}, C_{m_\alpha}$	Non-dimensional derivatives w.r.t. α, $\partial C_D/\partial \alpha$, etc.
$C_{D_\delta}, C_{l_\delta}, C_{L_\delta}, C_{m_\delta}, C_{n_\delta}, C_{Y_\delta}$	Non-dimensional derivatives w.r.t. δ, $\partial C_D/\partial \delta$, etc.
$C_{l_p}, C_{n_p}, C_{Y_p}$	Non-dimensional derivatives w.r.t. p, $\partial C_l/\partial(pb/2V_e)$, etc.
$C_{l_r}, C_{n_r}, C_{Y_r}$	Non-dimensional derivatives w.r.t. r, $\partial C_l/\partial(rb/2V_e)$, etc.
$C_{l_\beta}, C_{n_\beta}, C_{Y_\beta}$	Non-dimensional derivatives w.r.t. β, $\partial C_l/\partial \beta$, etc.
C_{L_q}, C_{m_q}	Non-dimensional derivatives w.r.t. q, $\partial C_L/\partial(q\bar{c}/2V_e)$, etc.
$C_{m_{\dot{\alpha}}}$	Non-dimensional derivative w.r.t. $\dot{\alpha}$, $\partial C_m/\partial(\dot{\alpha}\bar{c}/2V_e)$
$\mathbf{f}$	Aerodynamic force vector, Equation (1)
$\mathbf{g}$	Gust vector, Equation (1)
$\mathbf{g}_1, \mathbf{g}_2$	Longitudinal and lateral versions of $\mathbf{g}$
$\mathbf{G}(s)$	System transfer function matrix, Equation (2)
G_{ik}	Element i,k of $\mathbf{G}$
$G_{DV}, G_{D\alpha}, G_{D\delta}$	Aerodynamic transfer functions relating drag to V, α, δ respectively
$G_{lp}, G_{lr}, G_{l\beta}, G_{l\delta}$	Aerodynamic transfer functions relating rolling moment to p, r, β, δ respectively
$G_{LV}, G_{Lq}, G_{L\alpha}, G_{L\delta}$	Aerodynamic transfer functions relating lift to V, q, α, δ respectively

$G_{mV}, G_{mq}, G_{m\alpha}, G_{m\delta}$	Aerodynamic transfer functions relating pitching moment to V, q, α, δ respectively
$G_{np}, G_{nr}, G_{n\beta}, G_{n\delta}$	Aerodynamic transfer functions relating yawing moment to p, r, β, δ respectively
$G_{yp}, G_{yr}, G_{y\beta}, G_{y\delta}$	Aerodynamic transfer functions relating sideforce to p, r, β, δ respectively
G_{TV}	Transfer function relating thrust to V
I_x, I_y, I_z, I_{zx}	Aircraft moments and product of inertia
$\hat{I}_x, \hat{I}_z, \hat{I}_{zx}$	Non-dimensional inertias, $I_x/\rho_e S(b/2)^3$, etc.
$\hat{I}_y$	Non-dimensional inertia in roll, $I_y/\rho_e S(\bar{c}/2)^3$
K	Damper gain, Equation (10)
L	Turbulence length scale, Equation (22)
m	Aircraft mass
$n(s)$	Normal load factor, $\Delta L/mg$
p, q, r	Roll, pitch and yaw rate components in body axes
s	Laplace variable
S	Reference area
t	Time
t^*	Non-dimensionalising time scale, $\bar{c}/2V_e$
u, v, w	Velocity components in body axes
V	Airspeed
x, y, z	Co-ordinates in body axes
$\mathbf{x}(s)$	State vector
α	Angle of attack
α_T	Thrust vector angle w.r.t. reference body x-axis
β	Sideslip angle
γ	Climb angle
δ	Control surface angle
δ_1	Dummy variable, Equation (11)
Δ	Small increment
θ	Pitch angle
λ_1, λ_2	Gust wavelengths along x, y axes, Equation (19)
μ	Relative density parameter, $2m/\rho_e S\bar{c}$
ρ	Atmospheric density
σ	Standard deviation
Σ	Summation
τ_1, τ_2, τ_3	Control system time constants, Equation (10)
ϕ	Roll angle
$\mathbf{\Phi}_g, \mathbf{\Phi}_x$	Input (gust) and output power spectral density matrices, Equation (3)

$\Phi_{g_k g_l}$	General element of $\mathbf{\Phi}_g$
$\Phi_{u_g u_g}, \Phi_{v_g v_g}, \Phi_{w_g w_g}, \Phi_{p_g p_g}$	Specific, diagonal element of $\mathbf{\Phi}_g$
ψ	Yaw angle
ω	Frequency
Ω_1, Ω_2	x, y components of wave number vector

Subscripts

e	Refers to equilibrium or trim value
g	Refers to gust

Superscripts

T	Vector or matrix transpose
*	Complex conjugate

1. INTRODUCTION

In recent years the need has arisen at ARL from time to time for the calculation of rigid-body gust response characteristics of aircraft configurations. Such calculations are useful, for example, in assessing the platform stability of air vehicles in a turbulent atmosphere. Applications include comparisons between competing RPV designs and design requirements for a target designator. Similarly, estimates can be made for vehicle flight path dispersion or for gust loads on different parts of the vehicle. Further, Military Specifications (e.g. Ref. 1) call for calculation of aircraft response to atmospheric disturbances for assessment of flying qualities.

This Note documents a general gust response program which has been developed at ARL in order to meet such requirements. Although the program has been configured to produce response spectra due to isotropic homogeneous turbulence with a von Karman spectrum it is readily adaptable to more complex turbulence models including cross-correlations. On the other hand the program may simply be used to provide aircraft gust response transfer functions which may be used in calculating the response to a discrete gust. Most of the parameters describing the aircraft configuration, flight conditions and aerodynamics are read in as inputs thus facilitating comparisons between different aircraft configurations, investigation of effects of changes in aerodynamic parameters, control system operation, etc. Although limitations exist in the range of validity of the theory it may be assumed that its use in assessing relative trends extends over a rather wider range.

Section 2 of this Note summarizes the background theory and treats in more detail the aircraft and gust mathematical models including a discussion of the range of validity implied by the various approximations. This is followed in Section 3 by a description of the computer program and Section 4 provides examples of its application to two widely different vehicle configurations.

2. THEORY

The theoretical treatment in this section is derived largely from References 2 and 4. The basic mathematical approach is first described followed by a more detailed look at the two basic components, viz. the aircraft dynamics model and the treatment of the gust input.

2.1 Mathematical Model

The aircraft response to gust input can be treated as a small disturbance problem about a nominal trim state. Thus a linearised set of equations provides an adequate description of the motion. A general Laplace Transformed set of equations can be written as follows:

$$\mathbf{A}(s)\mathbf{x}(s) = \mathbf{f}(s) = \mathbf{B}(s)\mathbf{g}(s). \tag{1}$$

Where $\mathbf{x}(s)$ is the Laplace Transform of the aircraft state vector, $\mathbf{f}(s)$ is the Laplace Transform of the aerodynamic force vector due to the gusts and the matrix $\mathbf{A}(s)$ relating the two contains a description of the aircraft aerodynamic and inertial characteristics. The force vector, $\mathbf{f}(s)$, can be derived from a representation of the gust velocity field $\mathbf{g}(s)$ through $\mathbf{B}(s)$, which is a matrix of "gust transfer functions". These will be treated in more detail in 2.3 below.

From Equation (1) a matrix of Transfer Functions, $\mathbf{G}(s)$, relating the aircraft response, $\mathbf{x}(s)$, to the gust input, $\mathbf{g}(s)$, can be obtained:

$$\mathbf{x}(s) = \mathbf{G}(s)\mathbf{g}(s). \tag{2}$$

If $\mathbf{x}(s)$ is an $(n \times 1)$ vector and $\mathbf{g}(s)$ is an $(m \times 1)$ vector it follows that $\mathbf{G}(s)$ is an $(n \times m)$ matrix. A substitution of $i\omega$ for s (ω being frequency in radians/s) yields the Fourier Transform representation of the transfer function $\mathbf{G}(i\omega)$. If the response to a discrete gust is required then this can be done directly in the frequency domain through the use of $\mathbf{G}(i\omega)$ followed by a transformation back to the time domain. On the other hand, a spectral representation of the response can be obtained immediately from the relation:

$$\mathbf{\Phi}_x = \mathbf{G}^* \mathbf{\Phi}_g \mathbf{G}^T. \tag{3}$$

Where the matrix $\mathbf{\Phi}_g$ is gust input power spectral density matrix $(m \times m)$ and $\mathbf{\Phi}_x$ is the output power spectral density matrix $(n \times n)$. Normally the diagonal elements of $\mathbf{\Phi}_x$ are of interest and, for these, Equation (3) can be expanded as follows:

$$\Phi_{x_i x_i}(\omega) = \sum_{k=1}^{m} \sum_{l=1}^{m} G_{ik}^*(i\omega) \Phi_{g_k g_l}(\omega) G_{il}(i\omega) \tag{4}$$

where the subscripts refer to elements of the respective matrices. For the case of zero input cross spectra, i.e. no cross-correlations, Equation (4) can be further simplified to give

$$\Phi_{x_i x_i}(\omega) = \sum_{k=1}^{m} |G_{ik}(i\omega)|^2 \Phi_{g_k g_k}(\omega). \tag{5}$$

The integral of the output power spectrum with respect to ω gives the variance of the aircraft response about its nominal steady value, i.e.

$$\sigma^2_{x_i} = \int \Phi_{x_i x_i}(\omega) d\omega, \tag{5a}$$

where $\sigma^2_{x_i}$ is the variance of the x_i component of the state vector.

2.2 Aircraft Model

The use of linearised small disturbance equations allows the aircraft dynamics to be decoupled into Longitudinal and Lateral response sets, thereby reducing the size of the $\mathbf{A}(s)$ matrix (Eqn 1). The equations can be augmented with additional equations describing any flight control systems that may be present.

2.2.1 Aircraft Dynamics

The decoupled, non-dimensional, small-disturbance equations used are taken from Reference 2. The nominal reference state is, in general, steady flight at a climb angle of γ_e. The thrust vector is assumed to be at an angle α_T with respect to a reference set of axes fixed in the body. This set of reference axes is assumed to be initially aligned with the velocity vector, i.e. stability axes.

The Longitudinal equations are in wind axes with the state vector given by:

$$\mathbf{x}(s) = [\Delta V(s), \alpha(s), q(s), \Delta\theta(s)]^T \tag{6}$$

while the Lateral equations are in body axes and the state vector is:

$$\mathbf{x}(s) = [\beta(s), p(s), r(s), \phi(s)]^T. \tag{7}$$

The $\mathbf{A}(s)$ matrix for each of these cases is given in Appendix 1. Note that since non-dimensional equations are used, the Laplace Transform is with respect to non-dimensional time, $(2V/\bar{c})t$. Further, for generality, the aerodynamics characteristics are written as aerodynamic transfer functions even though in practice these are replaced by their quasi-steady approximations in terms of aerodynamic derivatives.

Although Equations (6) and (7) list only the state variables, responses for related variables can readily be derived. For example the variation in load factor, $\Delta n(s)$ in g's, in response to a vertical gust, w_g say, is related to $\alpha(s)$ and $\Delta V(s)$, the incidence and airspeed responses due to the vertical gust, respectively:

$$\Delta n(s) = \frac{C_{L_\alpha}}{C_{W_e}}\left(\alpha(s) - w_g(s)\right) + 2\Delta V(s). \tag{8}$$

Similarly, the yaw angle $\psi(s)$ is obtained from the integral of the yaw rate $r(s)$ by:

$$(\mathcal{R}) . s . \psi(s) = \sec \gamma_e . r(s) \tag{9}$$

where $\mathcal{R}$ is the ratio of span to mean aerodynamic chord.

2.2.2 Control Systems

The effect of automatic flight control systems on the gust response can be obtained by extending the state vectors, Equations (6) and (7), and the **A**(s) matrix to model the systems present. For example the pitch or yaw damper systems on the Mirage III may be modelled by the following equation:

$$(\tau_1 s + 1)(\tau_2 s + 1)(\tau_3 s + 1)^2 \delta(s) = K . s . q(s) \tag{10}$$

where $\delta(s)$ is the damper response to pitch rate, $q(s)$, K is a constant gain (depending on speed and height) and τ_1, τ_2, τ_3 are time constants. Thus, for example, the longitudinal state vector, Equation (6), need only be augmented by $\delta(s)$ while the **A**(s) matrix has an additional row, $[0, 0, -Ks, 0, h(s)]$, and an additional column, $[-G_{D\delta}, -G_{L\delta}, -G_{m\delta}, 0, h(s)]^T$, where $h(s) = (\tau_1 s + 1)(\tau_2 s + 1)(\tau_3 s + 1)^2$.

In practice, the computer program described in Section 3 allows only quadratic functions of s in the **A**(s) and **B**(s) matrices. This is readily accommodated by introducing an additional state variable, $\delta_1(s)$ say, and replacing Equation (10) with the two equations:

$$\begin{aligned} (\tau_2 s + 1)(\tau_3 s + 1)\delta(s) &= \delta_1(s) \\ (\tau_1 s + 1)(\tau_3 s + 1)\delta_1(s) &= Ksq(s) \end{aligned} \tag{11}$$

The equivalent **A**(s) matrix is shown in Appendix 1.

2.3 Gust Model

This section deals with the right-hand side of Equation 1. Different levels of approximation for the gust input are first described followed by a discussion of the gust spectra to be used. It is assumed throughout that the aircraft samples a *frozen gust field on a straight line, i.e. there is no* departure of the aircraft from rectilinear flight.

2.3.1 Point Approximation

The gust field is characterized by the three turbulent velocity components u_g, v_g, and w_g. In the point approximation the variations of these velocities over the aircraft are ignored, i.e. the aircraft is treated effectively as a point. Consistent with the small disturbance assumptions the gust vector, **g**, is separated into longitudinal and lateral sets, $\mathbf{g}_1$ and $\mathbf{g}_2$ respectively. Non-dimensionalising by dividing by V_e and noting that in the point approximation $w_g/V_e = \alpha_g$ and $v_g/V_e = \beta_g$, the two sets become:

$$\begin{aligned} \mathbf{g}_1 &= [u_g/V_e, \alpha_g]^T \\ \mathbf{g}_2 &= [\beta_g]^T \end{aligned} \tag{12}$$

It follows that the **B**(s) matrices (Eqn 1) required to relate these to aerodynamic forces are:

$$\mathbf{B}_1(s) = \begin{bmatrix} G_{DV}\ \ G_{TV} & G_{D\alpha} \\ G_{LV} & G_{L\alpha} \\ G_{mV} & G_{m\alpha} \\ 0 & 0 \end{bmatrix} ; \quad \mathbf{B}_2(s) = \begin{bmatrix} G_{Y\beta} \\ G_{l\beta} \\ G_{n\beta} \\ 0 \end{bmatrix} \tag{13}$$

The aerodynamic transfer functions, G_{TV}, G_{DV}, etc., are usually replaced with their quasi-steady approximations (see Appendix 1).

2.3.2 Power Series Method

This is an extension to the point approximation in that the gust field is now represented by the gust velocities and their first derivatives. It is still assumed that variations in the z-direction (normal to the flight direction) are negligible. The remaining two-dimensional velocity field is expanded as a Taylor series about the aircraft centre of gravity, and higher order terms are neglected, e.g.

$$w_g(x, y, t) = w_g(t) + \partial w_g/\partial x \, . \, x + \partial w_g/\partial y \, . \, y + \text{higher order terms.} \tag{14}$$

For aircraft rigid-body responses Reference 1 suggests that it is sufficient to consider only the uniform gust immersion terms u_g, v_g, w_g together with linear gradient terms

$$\begin{aligned} (1/V_e) \, . \, \partial w_g/\partial x &= q_g \\ (1/V_e) \, . \, \partial w_g/\partial y &= -p_g \\ (1/V_e) \, . \, \partial v_g/\partial x &= r_g \end{aligned} \tag{15}$$

Further, u_g, v_g, w_g and p_g can be considered to be mutually independent (uncorrelated) while correlations have to be accounted for between w_g and q_g, and between v_g and r_g. Now Reference 3 notes that, of the gradient terms, p_g is dominant: consequently only this term is retained in the following treatment, for simplicity.

The longitudinal gust set thus remains the same as in Equation (12) while the lateral set becomes:

$$\mathbf{g}_2 = [\beta_g, p_g]^T \tag{16}$$

where the p_g gust can be thought of as corresponding to a roll rate. The equivalent $\mathbf{B}_2(s)$ matrix (Eqn (13)) becomes:

$$\mathbf{B}_2(s) = \begin{bmatrix} G_{Y\beta} & \frac{b}{2} G_{Yp} \\ G_{l\beta} & \frac{b}{2} G_{lp} \\ G_{n\beta} & \frac{b}{2} G_{np} \\ 0 & 0 \end{bmatrix} \tag{17}$$

2.3.3 Range of Validity

Assuming rectilinear flight at speed V_e and neglecting variations in the z-direction the two-dimensional gust spectral component as experienced by the aircraft is a velocity field of the form

$$e^{i\Omega_1 V_e t} e^{i(\Omega_1 x + \Omega_2 y)} \tag{18}$$

which represents a time periodic velocity at any point (x, y) of the vehicle. Ω_1 and Ω_2 are related to the wavelengths λ_1 and λ_2, in the x and y directions, i.e.

$$\Omega_{1,2} = 2\pi/\lambda_{1,2}. \tag{19}$$

The range of validity of the point and power series (or linear field) approximations can be stated in terms of Ω_1 and Ω_2. Reference 5 notes that the linear field approximation gives a good representation of the gust velocity distribution over the aircraft only when Ω_1 and Ω_2 are "small enough", i.e. when λ_1 and λ_2 are considerably (e.g. ten times) larger than the length and span, respectively, of the aircraft.

Further, considerations from unsteady aerodynamics provide limits on Ω_1 and Ω_2 for which the quasi-steady approximation is acceptable. Thus the following approximate limits are obtained (Refs 2, 5):

$$\Omega_1 \, . \, \tfrac{1}{2}\bar{c} < 0 \cdot 1, \tag{20}$$

$$\Omega_2 \, . \, b < 2. \tag{21}$$

These are roughly compatible with the limitations on k_1, k_2 due to the linear field representation, implying a consistent use of quasi-steady aerodynamics with the linear field model. In general, the range of frequencies covered by such a treatment is adequate for rigid body motions appropriate to stability, guidance and control problems, but appreciable errors can occur if structural mode responses are important. For this case a more accurate aerodynamic theory should be employed and, consistent with this, a more exact spatial distribution of the gust field (see, for example, Ref. 5).

2.3.4 Gust Spectra

The simplest model of turbulence assumes that above the planetary boundary layer (approximately 2000 ft) the turbulence is isotropic and homogeneous. This means that length scales and intensities are the same in all directions. The von Karman spectra for the turbulence velocities are given by (Ref. 1):

$$\Phi_{u_g u_g}(\Omega_1) = \sigma_g^2 (2L/\pi)/[1 + (1.339 L\Omega_1)^2]^{5/6}, \tag{22}$$

$$\Phi_{v_g v_g}(\Omega_1) = \Phi_{w_g w_g}(\Omega_1) = \sigma_g^2 (L/\pi)[1 + \tfrac{8}{3}(1.339 L\Omega_1)^2]/[1 + (1.339 L\Omega_1)^2]^{11/6}, \tag{23}$$

where σ_g is the turbulence intensity and L the length scale, taken to be 2500 ft from Reference 1. All velocity cross spectra are zero in isotropic turbulence. However, cross spectra arise even in isotropic turbulence if variations in velocities over the vehicle are allowed (see, for example, Ref. 4). These are neglected here as explained in Section 2.3.2. The only other spectrum required here is that for p_g, given by Reference 1:

$$\Phi_{p_g p_g}(\Omega_1) = (\sigma_g^2/L)0.8(\pi L/4b)^{1/3} \Big/ \left\{1 + \left(\frac{4b}{\pi}\Omega_1\right)^2\right\} \tag{24}$$

where b is the span.

All spectra above are single sided so that the integration limits in Equation (5a) are from zero to infinity. Note that Ω_1 and frequency, ω, are related by $\Omega_1 V_0 = \omega$ (see Eqn (18)).

Although a simple isotropic model of turbulence has been described, the general approach used makes it relatively simple to use a more complex model appropriate to, say, flight at lower altitudes.

3. COMPUTER PROGRAM

The main program has been designed to be as flexible as possible to enable the user to alter it to suit the particular problem in hand. Thus the main program concentrates on defining the problem parameters while the basic calculations are achieved by calls to general purpose subroutines. It is also possible to use the program simply to generate transfer functions for any system of the form given by Equation 1. A general description of the program follows including a summary of the input requirements and output produced by the current gust response calculations. The program is written in Fortran and implemented on the ARL DEC System 10.

3.1 General Description

The basic structure of the main program GUSTR is shown in Figure 1. The problem dimensions (i.e. dimension of the **A** matrix) and all variable dimensions used in the program are defined at the beginning of the program in a Parameter statement. This is followed by a block with labelling and initialisation data required for later plotting of the results. The plot files for the Calcomp plotter are prepared via the program "TRANS" described in Reference 6.

The next step, constituting the problem definition, is to specify the elements of the $\mathbf{A}(s)$ and $\mathbf{B}(s)$ matrices (Eqn (1)). Each element of $\mathbf{A}(s)$ and $\mathbf{B}(s)$ is assumed to be a second order polynomial in s, i.e.

$$a_0 + a_1 s + a_2 s^2$$

or (25)

$$b_0 + b_1 s + b_2 s^2.$$

In order to define $\mathbf{A}(s)$, three arrays of coefficients are set up, the first to contain all the zero order coefficients (a_0, etc.), the second to contain the first order coefficients (a_1, etc.) and the third to contain the second order coefficients (a_2, etc.). Three two-dimensional arrays, $A0(I, J)$, $A1(I, J)$ and $A2(I, J)$ are used for this purpose such that $A0(I, J)$ contains the a_0 coefficient of the (I, J) element of $\mathbf{A}(s)$ etc. Similar considerations apply to $\mathbf{B}(s)$ except that, since transfer functions are calculated for one gust component at a time, one dimensional arrays $B0(I)$, $B1(I)$ and $B2(I)$ are required.

For the aircraft case the elements of the $\mathbf{A}(s)$ matrices have been listed in Appendix 1 and the equivalent $\mathbf{B}(s)$ matrices are given by Equations (13) and (17).

Having defined the system matrices the solution proceeds with a call to subroutine "TRANSF", which calculates the polynomial coefficients of the denominator and numerators of each of the transfer functions defined by the system. The denominator, or characteristic polynomial, roots are obtained by a call to subroutine "POLRT". Polynomial coefficients of any extra transfer functions required can be calculated at this stage. The present program calculates those for the normal acceleration at the centre of gravity (Eqn (8)) in the Longitudinal case and the yaw angle (Eqn (9)) in the Lateral case.

The frequency response functions, $G_{ik}(i\omega)$, are obtained by substituting $s = i\omega t^* = i\Omega\bar{c}/2$, where the non-dimensionalising time, $t^* = \bar{c}/2V_e$, appears because the Laplace variable s is with respect to non-dimensional time. The response power spectral densities then follow from Equation (5) using the spectra given in Section 2.3.4. In making these calculations use is made of subroutine "CPOL" which evaluates a polynomial having complex coefficients. "CPOL" is appended to the main program.

Computation time is determined largely by subroutine "TRANSF". If $\mathbf{A}(s)$ is $n \times n$ then "TRANSF" has to evaluate $(n + 1)$ determinants. Thus CPU time is proportional to $n!n(n + 1)$. For $n = 6$ this amounts to approximately 60 seconds on the DEC System 10, increasing very rapidly for larger n. If larger order systems need to be treated, computation time can be reduced by allowing higher order polynomials as elements of $\mathbf{A}(s)$ and $\mathbf{B}(s)$. This would require an alteration to subroutine "TRANSF".

3.2 Inputs

The data required to define the matrices $\mathbf{A}(s)$ and $\mathbf{B}(s)$ will depend on the particular problem in hand. In general the data can be divided into three groups related to aircraft configuration, flight conditions and aerodynamics. As an example Appendix 2 lists the set of data required to calculate longitudinal and lateral gust responses of the Mirage. The pitching and yawing moment data are often given relative to a reference centre of gravity position and the program adjusts these to the actual c.g. The drag due to pitch damper derivative, C_{D_q}, has been set to zero to avoid a non-linearity arising from its sign depending on the sign of α. It is in any case small. It has also been assumed that the reference state is one of level flight, $\gamma_e = 0$, and that the thrust line is parallel to the reference axes, i.e. $\alpha_T = 0$.

With the given data, the program carries out all the necessary non-dimensionalising and auxiliary calculations necessary to set the matrix elements. One additional piece of information required is the thrust due to speed derivative, C_{T_V}. For a jet powered aircraft such as the Mirage, constant thrust propulsion can be assumed so that $C_{T_V} = -2C_{D_e}$.

Having set the elements of $\mathbf{A}(s)$ and $\mathbf{B}(s)$ and calculated the transfer function elements, G_{ik}, the program only needs the turbulence length scale to define the input spectra σ_g^2 (i.e. from Eqns (22)–(24)) and obtain the corresponding output spectra σ_g^2 from Equation (5), where σ_g^2 is the relevant gust variance.

3.3 Ouputs

The choice of output depends on the application and can be readily changed by the user. Immediately available are the coefficients of the Laplace form of the transfer functions, $G_{ik}(s)$, the characteristic roots of the system, the real and imaginary parts of the transfer functions as functions of frequency, $G_{ik}(\Omega_1)$, and output spectra as functions of frequency, $\Omega_1 = \omega/V_e$.

For plots of frequency response or output spectrum against frequency it is generally convenient to use a log scale for the abscissa because of the wide frequency range generally of interest. In this case it is useful to plot spectral density multiplied by frequency, i.e. $\Omega_1\Phi(\Omega_1)$ rather than $\Phi(\Omega_1)$, as the ordinate since the area under such a graph is then proportional to the variance (Eqn (5a)):

$$\int_0^\infty \Omega_1\Phi(\Omega_1)d(\log_{10}\Omega_1) = \int_0^\infty \Omega_1\Phi(\Omega_1)d(\ln\Omega_1)/\ln 10 = (1/\ln 10)\int_0^\infty \Phi(\Omega_1)d\Omega_1 = \sigma^2/\ln 10$$

Thus

$$\sigma^2 = 2\cdot 3\int_0^\infty \Omega_1\Phi(\Omega_1)d(\log_{10}\Omega_1). \tag{26}$$

4. EXAMPLES

Typical results are shown for two widely differing configurations in this section. The first one can be thought of as a possible RPV configuration while the second one relates to the Mirage III. The usefulness of the program in assessing the effects on gust response of changes in c.g. position, control systems and aerodynamic parameters is demonstrated.

Two graphs are plotted for each response variable of interest. The upper graph plots $\log_{10}$ of the square of the frequency response and the lower graph plots the output power spectrum in the form $\Omega_1\Phi(\Omega_1)/\sigma_g^2$, where σ_g^2 is the appropriate gust variance. In each case the abscissa is $\log_{10}(\Omega_1)$ where $\Omega_1 = \omega/V_e$ (V_e in ft/s here). Since the present program defines the input spectra to be half those given by Equations (22)–(24) the output variances are 4·6 times the area under the output spectra graphs (Eqn (26)).

4.1 RPV Configuration

Results shown in Figures 2–4 are for a vehicle of chord 0·11 m (0·36 ft) and aspect ratio 10 flying at 106 m/s (348 ft/s) at an altitude of 305 m (1000 ft). For this case different turbulence length scales are used for the vertical gusts ($L = 305$ m in w_g and p_g spectra) and the longitudinal and lateral gusts ($L = 561$ m in u_g and v_g spectra).

Figure 2 shows the pitch rate, q, response to vertical gust, w_g, as a function of static margin. As the c.g. moves forward the static margin increases from a low value of 0·1 to a maximum of 1·47. At the same time the vehicle becomes much more sensitive to vertical gusts. The range of validity of the calculations (Eqn (20)) is also indicated on Figure 2. Equivalent results for the pitch attitude, θ, response are shown in Figure 3. The two obvious sets of peaks correspond to the short period and Phugoid frequencies of the vehicle. For static margin of 0·1 there is very little pitch attitude response for $\log_{10}\Omega_1$ above -2 (i.e. ω above 3·48 rad/s) in contrast to a considerable amount of energy between $-2 < \log\Omega_1 < -1\cdot5$ ($3\cdot48 < \omega < 11\cdot00$ rad/s) for a static margin of 1·47. In this latter case the area under the curve between these limits is, very approximately, 1·0 (square inches) $\times 5\times10^{-7}$ (scale) which contributes to a variance $\sigma^2/\sigma_g^2 = 4\cdot6\times5\times10^{-7} = 2\cdot3\times10^{-6}$ (rad^2/(ft s^{-1})^2). For a gust intensity, σ_g, of 10 ft/s this would mean a pitch attitude response standard deviation of 0·015 rad = 0·87 degrees in the given frequency range.

An example of roll rate, p, response to lateral gust, v_g, and gust gradient, p_g, is shown in Figure 4. The peaks near $\log(\Omega_1) = -2$ correspond to the Dutch Roll frequency. The roll rate gust response variances due to v_g and p_g can be obtained from the areas under the respective output spectra over the desired frequency range, as was done above with the pitch attitude case. Since it has been assumed that there is no cross correlation between the v_g and p_g gusts the total output variance for roll rate is given by the sum of the variances due to v_g and to p_g.

4.2 Mirage III Configuration

Figures 5–8 show results for roll rate and yaw rate responses of a Mirage type configuration to lateral gusts, v_g. These are for flight at $M = 0{\cdot}9$ at 610 m (2000 ft) altitude. The turbulence length scale has been taken as 2500 ft.

Figures 5 and 6 show the influence of dihedral effect, C_{l_β}, on the roll rate and yaw rate responses respectively. The Dutch Roll frequency (about 4 rad/s), and the roll mode time constant (about 0·25 s) do not depend strongly on C_{l_β}, but the spiral mode time constant changes from about 400 s for C_{l_β} = 0·015 (close to the true value) to a very low 6 s for C_{l_β} = 0·15. The effect on the roll response to lateral gust is also very large, as may be expected (Fig. 5), but the yaw rate response only changes slightly.

Finally, the effect of yaw damper on roll and yaw rate responses is shown in Figures 7 and 8. The damper is clearly effective around the Dutch Roll frequency and consequently is also effective in diminishing the roll and yaw gust responses which are predominantly at this frequency. The approximate range of validity shown in Figure 8 covers most of the output spectrum even for this case of large chord. The range of validity on Ω_1 can be extended by a factor of 10 if unsteady oscillatory aerodynamics are used.

Similar plots can readily be produced for any other variables of interest including any of the state variables (Eqns (6) and (7)) or extra variables such as normal acceleration, yaw angle, etc. (e.g. Eqns (8) and (9)). If only the responses to the gust components given by Equations (12) and (16) are considered then cross-correlations can be neglected and Equation (5) can be used to give the output spectra. The program can be quite easily extended to include other gust components: cross-correlations would require the use of the more general Equation (4) for the output spectra.

5. CONCLUSIONS

This Note has described a Fortran program developed for aircraft rigid-body gust response calculations. The theoretical basis and its limitations have been summarised, with a more detailed description of the aircraft and gust models and of the program organisation.

Decoupled small disturbance equations of motion have been used with provision for additional equations for any flight control systems. Most Configuration, Flight Condition and Aerodynamic Data are read in as inputs and can readily be varied. The gust spectra described are appropriate to isotropic homogeneous turbulence and cross-correlations have not been included in the examples given.

The main program has been organized in a flexible form to allow easy alteration either to the mathematical model or the gust spectra. Thus, for example, changes in flight control systems or addition of extra output variables can be readily accommodated. Similarly, it is relatively easy to modify the program to cater for cross-correlations in the gust inputs. The calculation of the gust response transfer functions is independent and separate from the evaluation of the output spectra so that it is possible to use the program simply to calculate the transfer functions of any linear system.

The examples given demonstrate the usefulness of the program in assessing the effects of variations in configuration, aerodynamics or control systems on the gust response of a given vehicle.

ACKNOWLEDGMENT

Mr P. Gottlieb provided the subroutine "TRANSF" which is an important component of the current program and contributes to its flexibility. His help in developing the general structure of the program is gratefully acknowledged.

REFERENCES

1. "Military Specification—Flying Qualities of Piloted Aircraft." MIL-F-8785C, November, 1981.
2. Etkin, B. *Dynamics of Atmospheric Flight*, John Wiley and Sons, Inc., 1972.
3. Houbolt, J. C. "Survey on Effect of Surface Winds on Aircraft Design and Operation and Recommendations for Needed Wind Research." NASA CR-2360, December 1973.
4. Etkin, B. "Theory of the Flight of Airplanes in Isotropic Turbulence Review and Extension." AGARD Report 372, April 1961.
5. Etkin, B. "The Turbulent Wind and its Effect on Flight." UTIAS Review No. 44, August 1980.
6. Nankivell, P. G., and Gilbert, N. E. "A General Purpose Output Program for use in Simulation." ARL Aero. Note 367, December 1976.

APPENDIX 1

Aircraft Dynamic Model

(i) *Basic Model*

The Longitudinal $\mathbf{A}(s)$ matrix is

$$\mathbf{A}_1(s) = \begin{bmatrix} (G_{TV}\cos\alpha_T - G_{DV} + 2C_{u_e}\sin\gamma_e - 2\mu s) & (C_{L_e} - G_{D_\alpha}) & 0 & C_{u_e}\cos\gamma_e \\ (G_{TV}\sin\alpha_T + G_{LV} + 2C_{u_e}\cos\gamma_e) & (G_{L\alpha} + C_{D_e} + 2\mu s) & (2\mu - G_{Lq}) & C_{u_e}\sin\gamma_e \\ -G_{mV} & G_{m\alpha} & (G_{mq} - \hat{I}_y s) & 0 \\ 0 & 0 & 1 & s \end{bmatrix} \quad \text{(A1)}$$

The lateral $\mathbf{A}(s)$ matrix is

$$\mathbf{A}_2(s) = \begin{bmatrix} (G_{y\beta} - 2\mu s) & G_{yp} & G_{yr} - 2\mu(\mathcal{R}) & C_{u_e}\cos\gamma_e \\ G_{l_\beta} & (G_{lp} - (\mathcal{R})\hat{I}_x s) & (G_{lr} + (\mathcal{R})\hat{I}_{zx} s) & 0 \\ G_{n_\beta} & (G_{np} + (\mathcal{R})\hat{I}_{zx} s) & (G_{nr} - (\mathcal{R})\hat{I}_z s) & 0 \\ 0 & 1 & \tan\gamma_e & (\mathcal{R})s \end{bmatrix} \quad \text{(A2)}$$

The aerodynamic transfer functions, G_{TV}, G_{DV}, etc., are usually replaced by a "quasi-steady" aerodynamic derivative representation, i.e.

$$\begin{aligned} G_{TV} &= C_{T_V} \\ G_{DV} &= C_{D_V} \\ &\vdots \\ G_{n_\beta} &= C_{n_\beta} \\ &\text{etc.} \end{aligned}$$

but

$$G_{m\alpha} = C_{m_\alpha} + sC_{m_{\dot\alpha}}$$

(ii) *Model with Pitch and Yaw Damper*

The yaw damper equations are (Eqn (11)):

$$\begin{aligned} (\tau_2\tau_3 s^2 + (\tau_2 + \tau_3)s + 1)\delta(s) &= \delta_1(s) \\ (\tau_1\tau_3 s^2 + (\tau_1 + \tau_3)s + 1)\delta_1(s) &= Ksq(s) \end{aligned} \quad \text{(A3)}$$

The state vector becomes (Eqn (6)):

$$\mathbf{x} = [\Delta V(s), \alpha(s), q(s), \Delta\theta(s), \delta_1(s), \delta(s)]^T \quad \text{(A4)}$$

with a similar extension for the lateral case (Eqn (7)).

The longitudinal matrix is augmented to

$$\underset{(6\times 6)}{\mathbf{A}(s)} = \left[\begin{array}{cccc:cc} & & & & 0 & G_{D\delta} \\ & \underset{(4\times 4)}{\mathbf{A}_1(s)} & & & 0 & G_{L\delta} \\ & & & & 0 & G_{m\delta} \\ & & & & 0 & 0 \\ \hdashline 0 & 0 & Ks & 0 & (\tau_2\tau_3 s^2 + (\tau_2 + \tau_3)s + 1) & 0 \\ 0 & 0 & 0 & 0 & 1 & (\tau_1\tau_3 s^2 + (\tau_1 + \tau_3)s + 1) \end{array}\right]$$

with a similar extension in the lateral case except that $G_{D\delta}$, $-G_{L\delta}$ and $-G_{m\delta}$ are replaced by $G_{Y\delta}$, $G_{l\delta}$ and $G_{n\delta}$ respectively.

APPENDIX 2

Input to Gust Response Program [Mirage]

(A) *Configuration Data*

1. Mass, m
2. Inertias
   - (i) Roll, I_x
   - (ii) Pitch, I_y
   - (iii) Yaw, I_z
   - (iv) Cross, I_{zx}
3. Reference Area, S
4. Reference Chord, $\bar{c}$
5. Reference Span, b
6. Centre Line Chord
7. C.G. Position as % of Centre Line Chord
8. Damper Gain (Eqn (10)), K

(B) *Flight Conditions*

9. Airspeed, V
10. Altitude, h, or Density, ρ
11. Turbulence Length Scale, L

(C) *Aerodynamics — Longitudinal*

12. C_{D_e}, Trim Drag
13. C_{D_V}, Drag due to speed
14. C_{D_α}, Drag due to incidence
15. C_{L_V}, Lift due to speed
16. C_{L_α}, Lift due to incidence
17. C_{L_q}, Lift due to pitch rate
18. C_{m_V}, Pitching Moment due to speed
19. C_{m_α}, Pitching Moment due to incidence
20. $C_{m_{\dot{\alpha}}}$, Pitching Moment due to rate of change of incidence
21. C_{m_q}, Pitching Moment due to pitch rate
22. C_{L_δ}, Lift due to pitch damper
23. C_{m_δ}, Pitching Moment due to pitch damper

(D) *Aerodynamics - Lateral*

24. C_{y_β}. Sideforce due to sideslip
25. C_{y_p}. Sideforce due to roll rate
26. C_{y_r}. Sideforce due to yaw rate
27. C_{l_β}. Rolling Moment due to sideslip
28. C_{l_p}. Rolling Moment due to roll rate
29. C_{l_r}. Rolling Moment due to yaw rate
30. C_{n_β}. Yawing Moment due to slideslip
31. C_{n_p}. Yawing Moment due to roll rate
32. C_{n_r}. Yawing Moment due to yaw rate
33. C_{y_δ}. Sideforce due to yaw damper
34. C_{l_δ}. Rolling Moment due to yaw damper
35. C_{n_δ}. Yawing Moment due to yaw damper

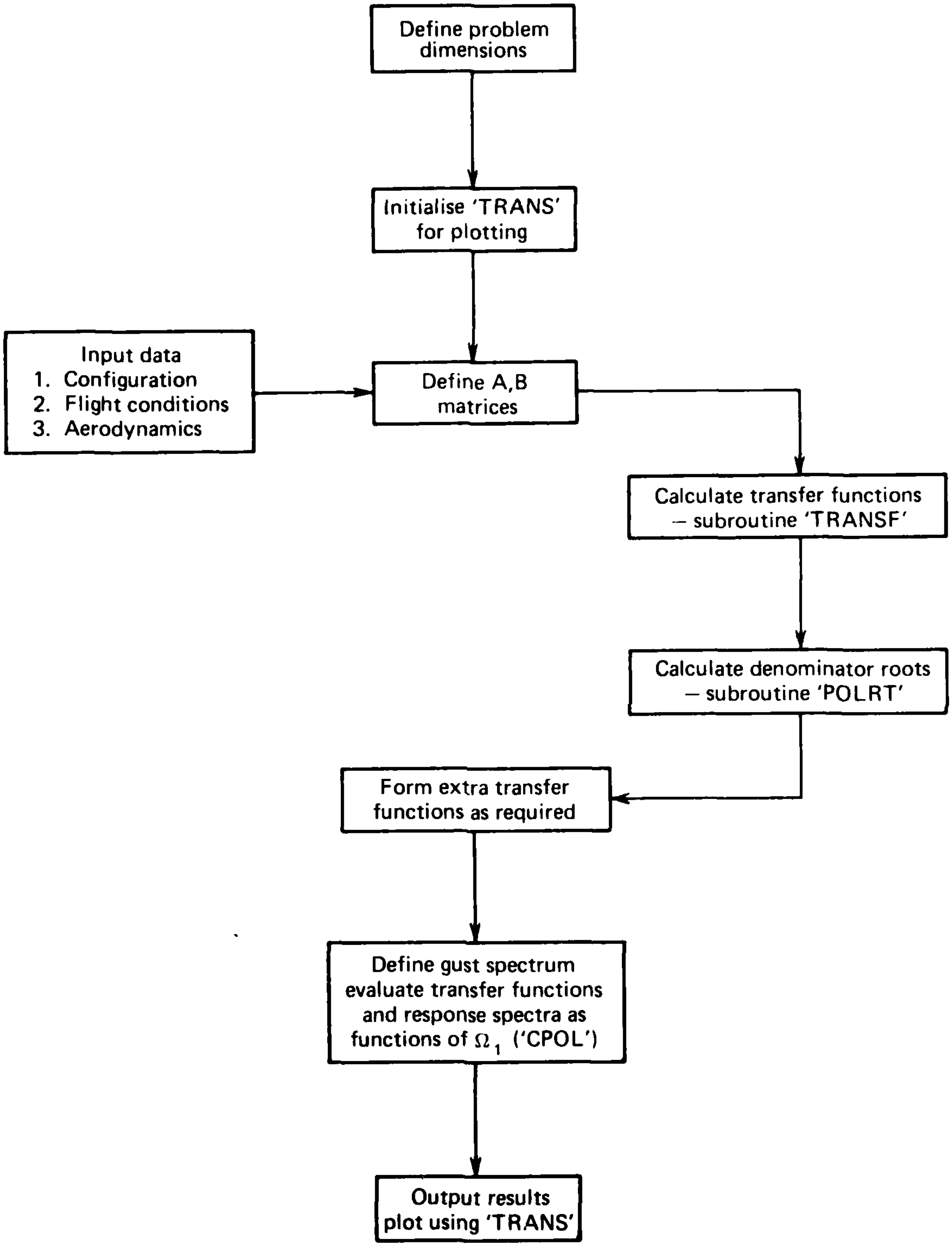


FIG. 1 BASIC STRUCTURE OF 'GUSTR'

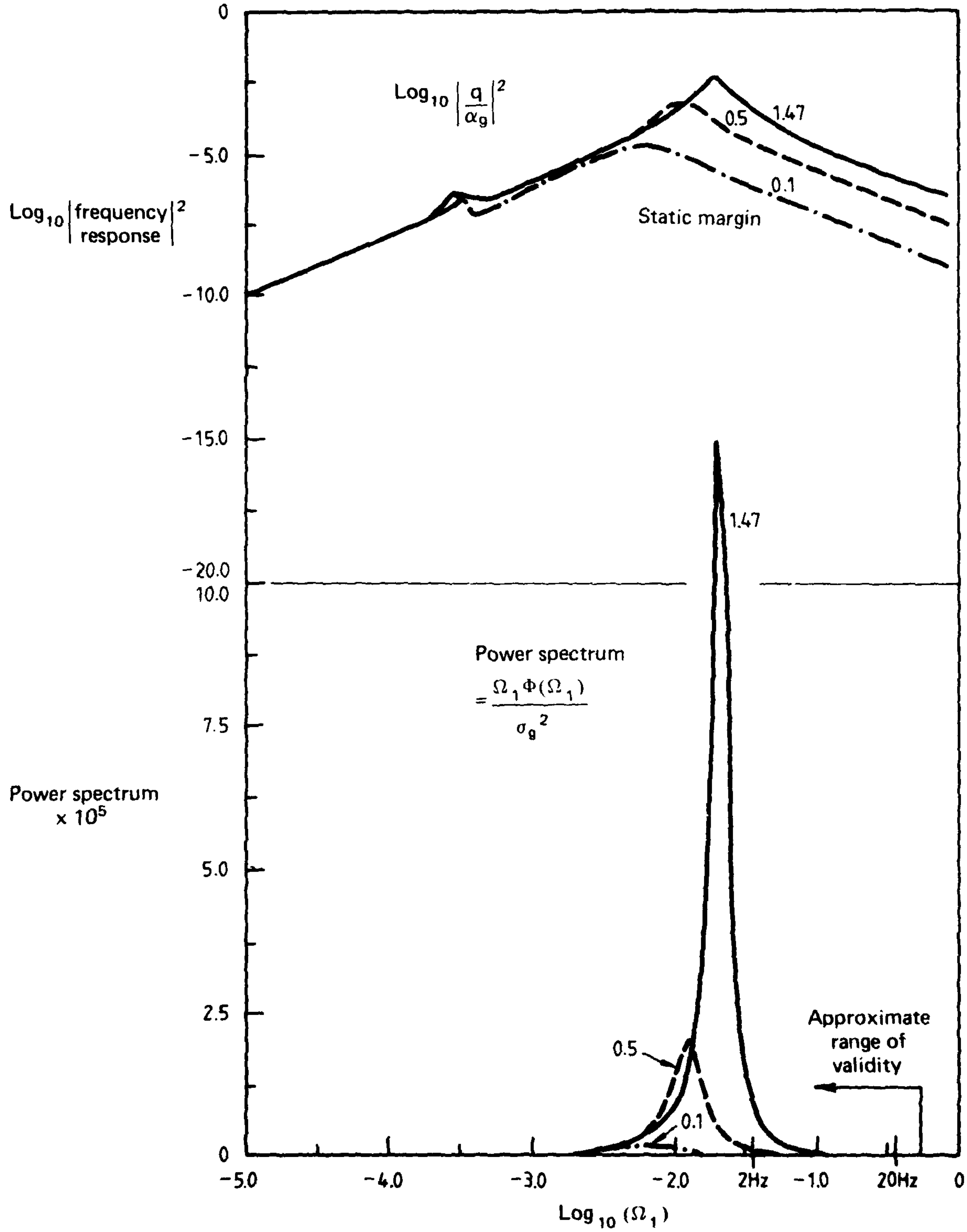


FIG. 2 PITCH RATE RESPONSE TO VERTICAL GUST, α_g – RPV

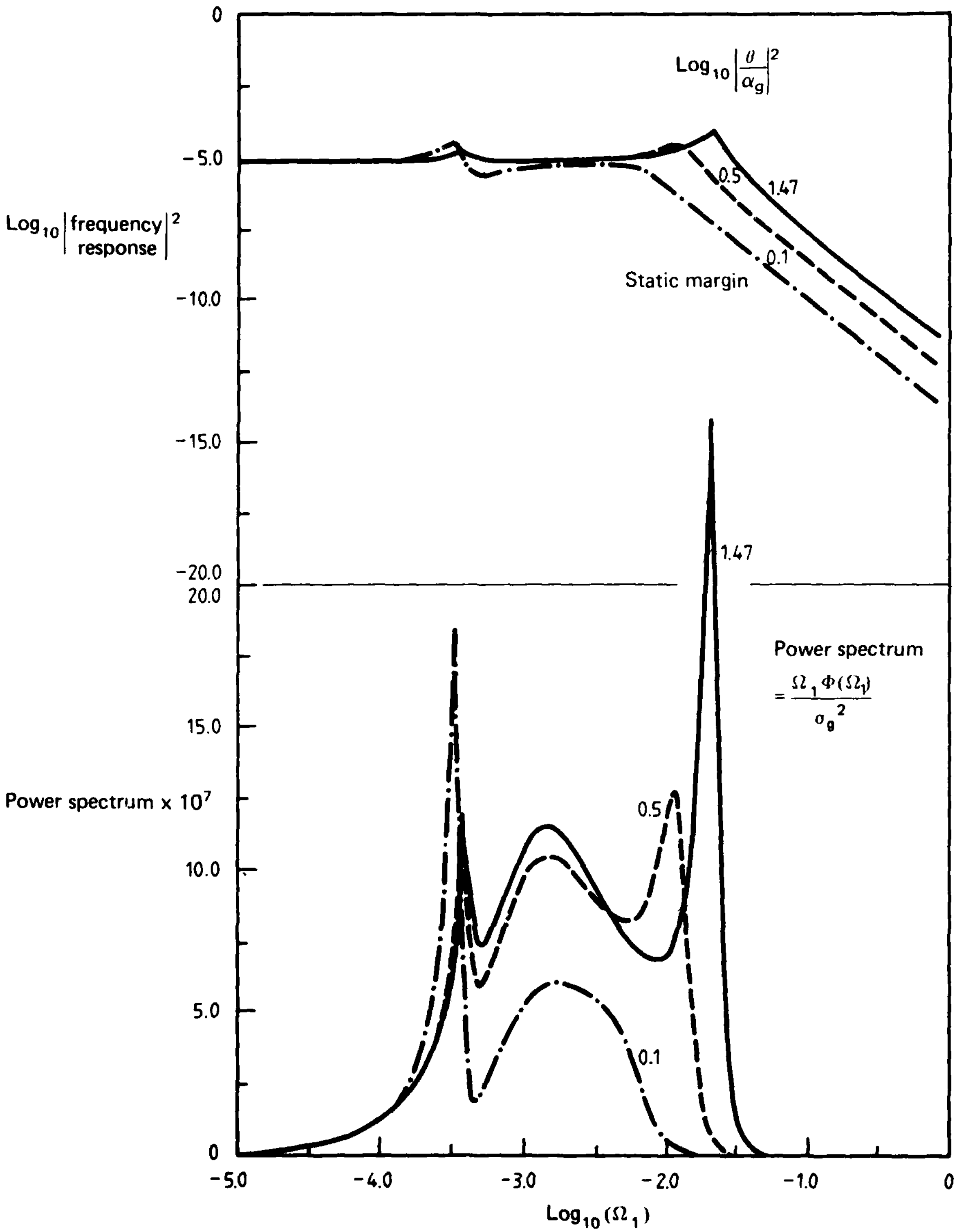


FIG. 3 PITCH ATTITUDE RESPONSE TO VERTICAL GUST, α_g – RPV

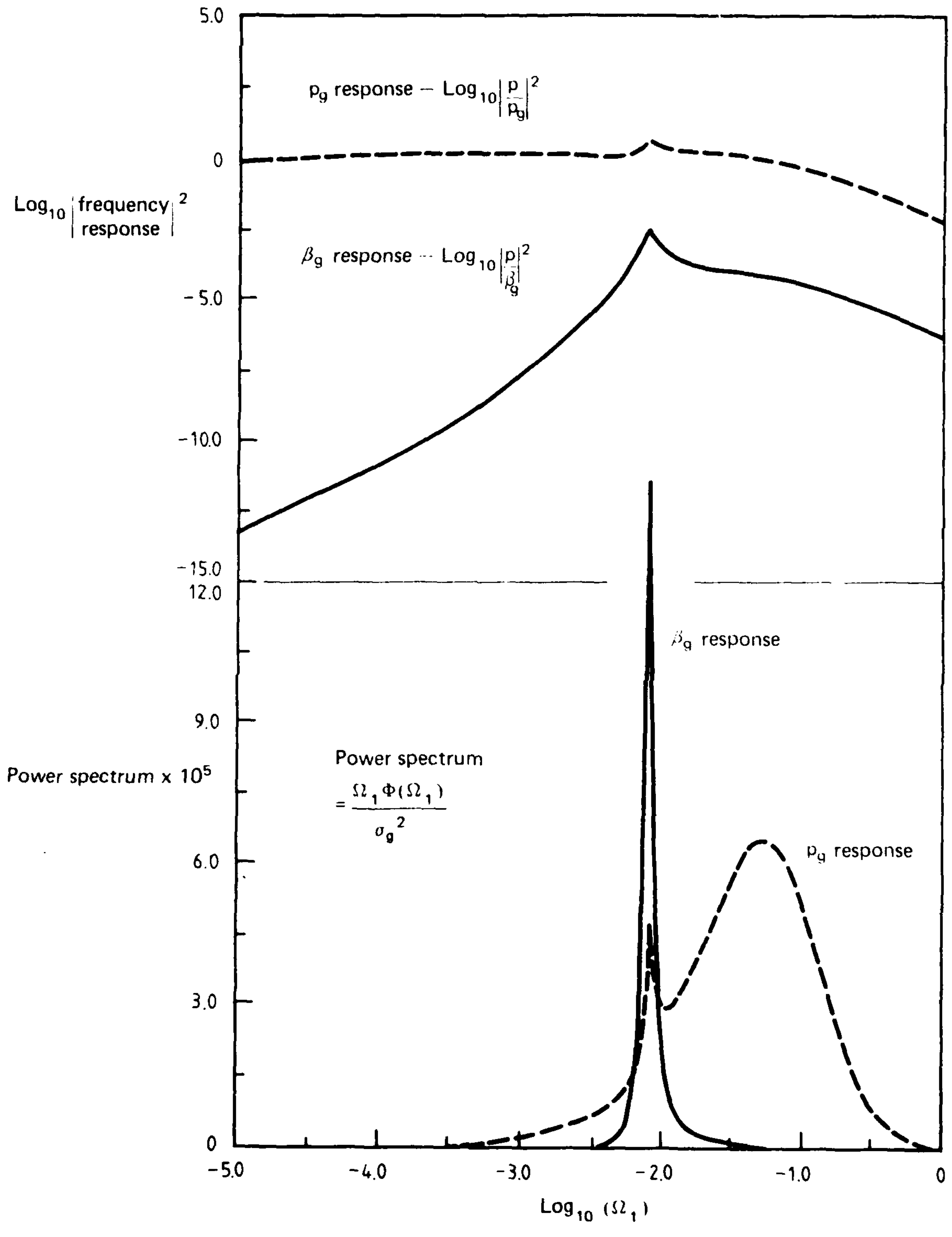


FIG. 4 ROLL RATE RESPONSE TO LATERAL GUST, β_g AND GUST GRADIENT, p_g — RPV

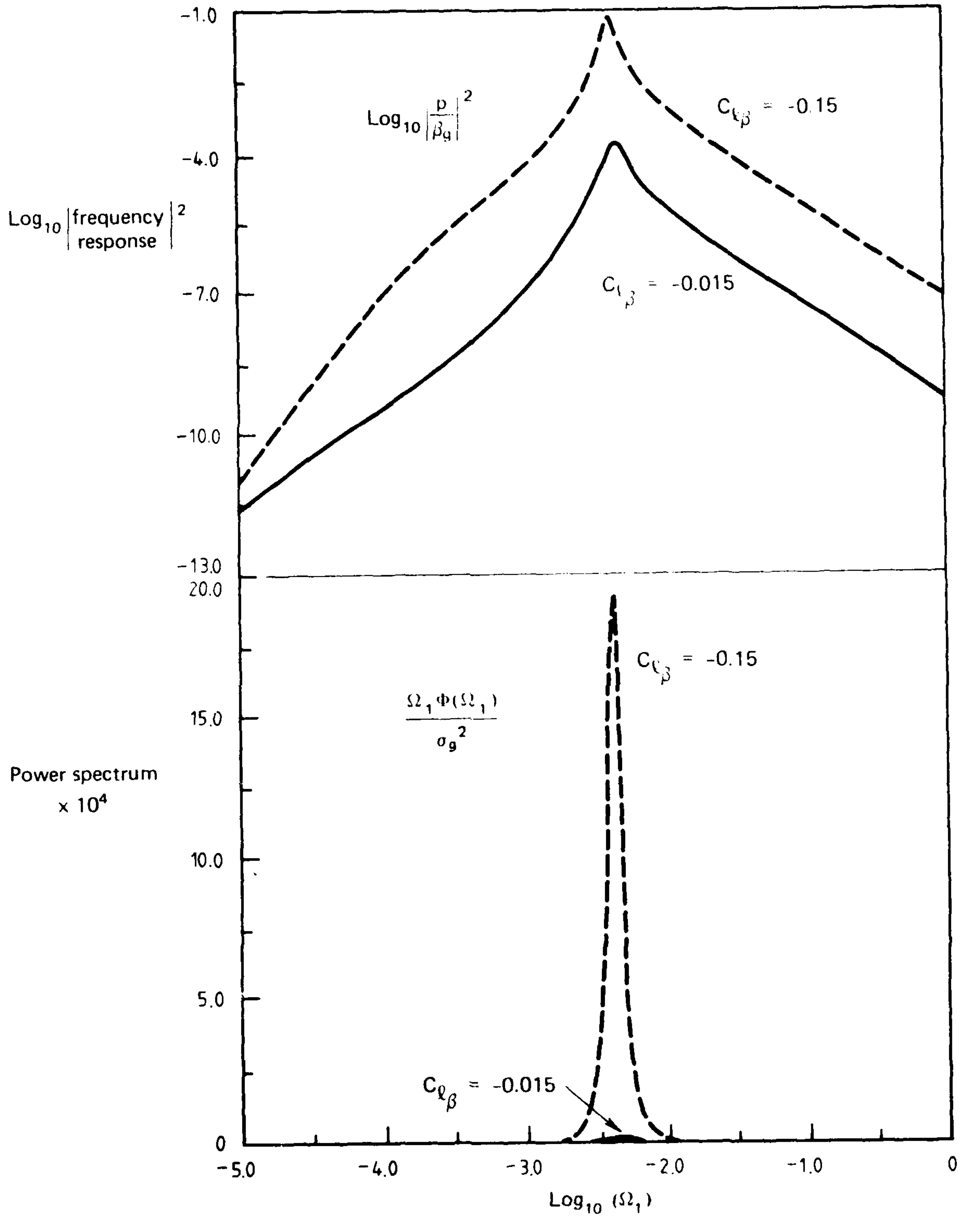


FIG. 5 ROLL RATE RESPONSE TO LATERAL GUST, β_g – MIRAGE
(YAW DAMPER OFF)

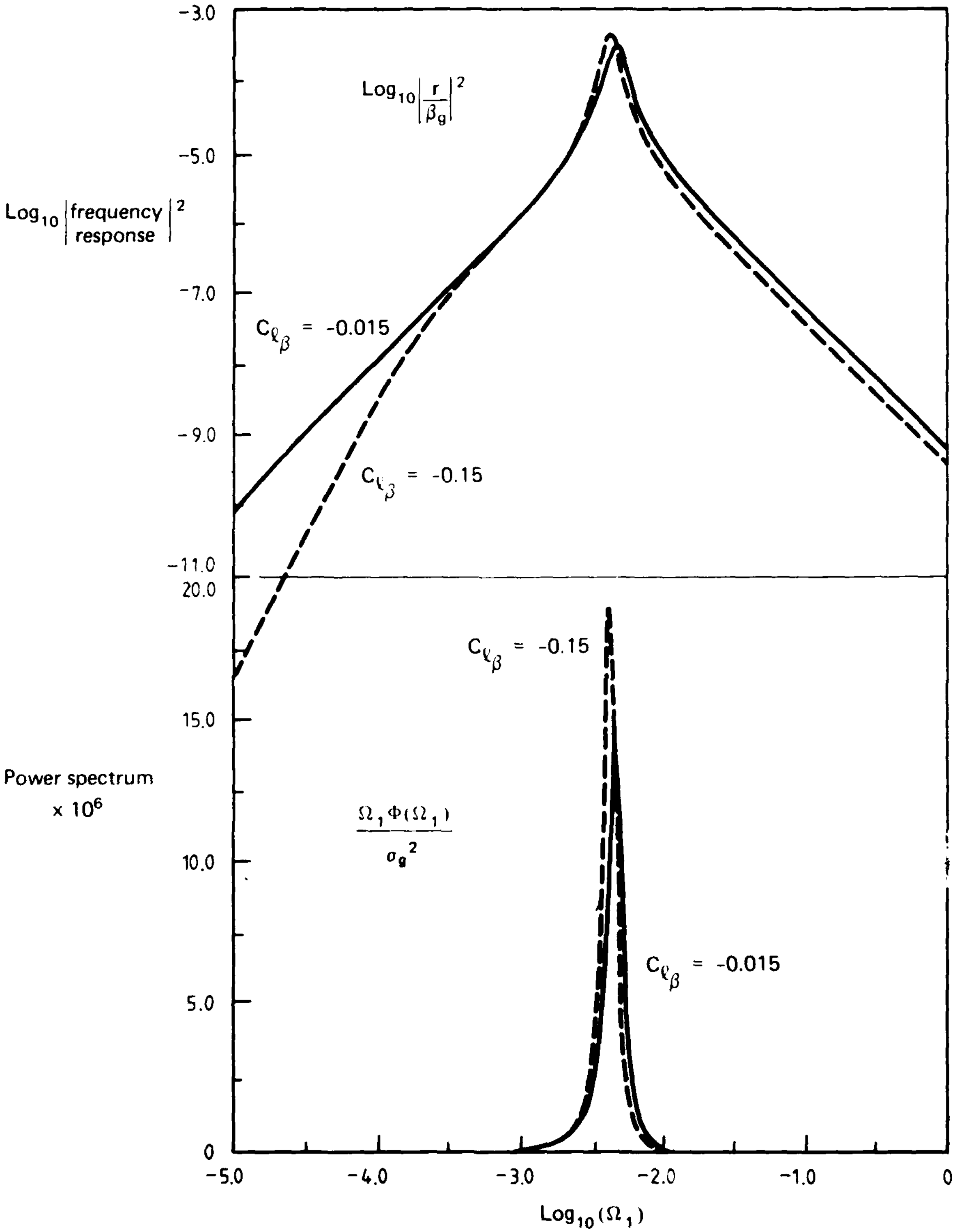


FIG. 6 YAW RATE RESPONSE TO LATERAL GUST, β_g – MIRAGE (YAW DAMPER OFF)

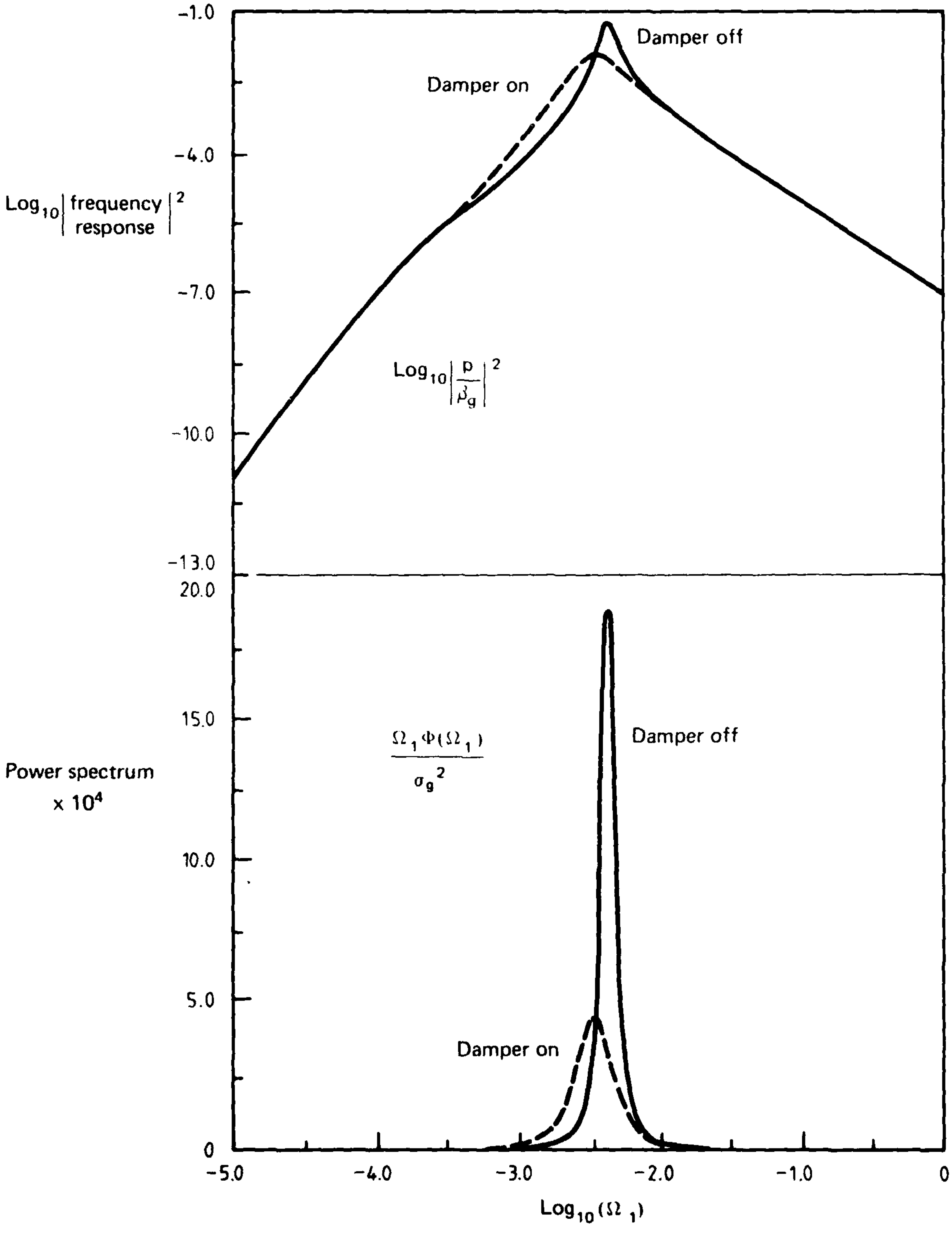


FIG. 7 ROLL RATE RESPONSE TO LATERAL GUST, β_g – MIRAGE
(C_{ℓ_β} = -0.15)

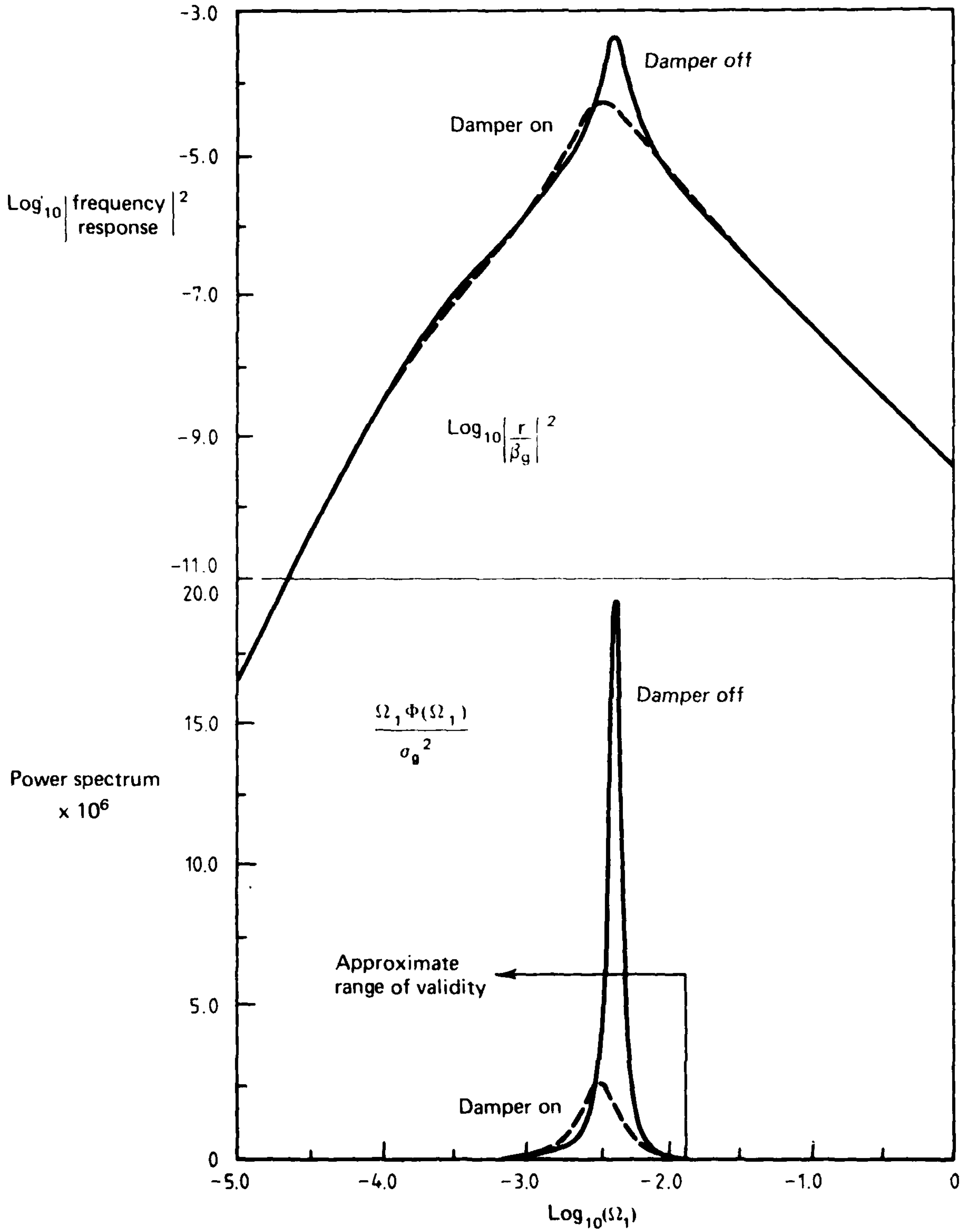


FIG. 8 YAW RATE RESPONSE TO LATERAL GUST, β_g – MIRAGE
(C_{ℓ_β} = -0.15)

DISTRIBUTION

	Copy No.
AUSTRALIA	
Department of Defence	
Central Office	
Chief Defence Scientist	1
Deputy Chief Defence Scientist	..
Superintendent, Science and Technology Programmes	..
Controller, Projects and Analytical Studies	..
Australian Defence Scientific and Technical Representative (UK)	-
Counsellor, Defence Science (USA)	-
Defence Central Library	2
Document Exchange Centre, DISB	3 19
Joint Intelligence Organisation	20
Librarian, H Block, Victoria Barracks, Melbourne	21
Aeronautical Research Laboratories	
Chief Superintendent	22
Library	23
Superintendent Aerodynamics	24
Divisional File- Aerodynamics	25
Author: R. A. Feik	26
D. A. Secomb	27
C. A. Martin	28
D. C. Collis	29
G. Long	30
C. K. Rider	31
D. J. Sherman	32
Materials Research Laboratories	
Chief Superintendent/Library	33
Defence Research Centre	
Library	34
Central Office	
Director-General—Army Development (NSO)	35 38
Defence Industry and Material Policy, FAS	39
Central Studies Establishment	
Information Centre	40
Engineering Development Establishment	
Library	41
Navy Office	
Navy Scientific Adviser	42
Army Office	
Army Scientific Adviser	43

Air Force Office		
Aircraft Research and Development Unit		
Scientific Flight Group		44
Library		45
Air Force Scientific Adviser		46
Technical Division Library		47
Director General Aircraft Engineering Air Force		48
HQ Support Command [SENGSO]		49
Department of Industry and Commerce		
Government Aircraft Factories		
Manager		50
Library		51
Statutory and State Authorities and Industry		
Commonwealth Aircraft Corporation, Library		52
Hawker de Havilland Aust. Pty. Ltd., Bankstown, Library		53
Universities and Colleges		
Melbourne	Engineering Library	54
Sydney	Engineering Library	55
New South Wales	Physical Sciences Library	56
RMIT	Library	57
CANADA		
Universities and Colleges		
Toronto	Institute for Aerospace Studies, Prof. B. Etkin	58
FRANCE		
ONERA, Library		59
UNITED KINGDOM		
British Library, Lending Division		60
UNITED STATES OF AMERICA		
NASA Scientific and Technical Information Facility		61
Spares		62–71